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A NEW CLASS OF DISTRIBUTED-ELEMENT MODELS FOR CYCLIC PLASTICITY—I. THEORY AND APPLICATION

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Abstract—A class of multi-dimensional Distributed-Element Models is proposed for constitutive modeling in cyclic plasticity. A new formulation in the "invariant-yield-surface" space is presented in which no kinematic hardening rule is needed to account for the subsequent yielding and strain hardening behavior. This physically-motivated class of models is not only mathematically simple but also parsimonious in parameters. Validity of the model is confirmed by comparison with experimental results from the literature.

1. INTRODUCTION

Structures of simple configurations and homogeneous materials may usually be approximated by simplified analytical models so that their response to complicated external loading can be analysed in some efficient way. When such simplified models are used for structural systems, the hysteretic response is often described by its overall interstory force–deflection relationship so as to avoid complex stress–strain calculations for which constitutive equations governing material behavior at a point are needed (Jayakumar, 1987). Although overall planar force–deflection representations in nonlinear structural analysis can reflect behavior of structural members or substructures as a whole, including both material and geometrical effects, they are not suitable for describing local response behavior in the case of complex mechanical systems or complicated loading conditions in which responses in different directions may interact significantly with one another. For that purpose, one needs to introduce appropriate constitutive laws depicting stress–strain relations at different material points, from which local response behavior can then be derived.

The one-dimensional Distributed-Element Model (DEM), introduced by Iwan (1966) for structural dynamic analysis, consists of a set of N elements connected in parallel, each of which consists of a linear spring with "stiffness" E_i in series with a slip element (Coulomb damper) of strength σ_i^*, as shown in Fig. 1. Each element in the assemblage is thus an ideal elasto–plastic element that has a force-deflection (uniaxial stress–strain) behavior as

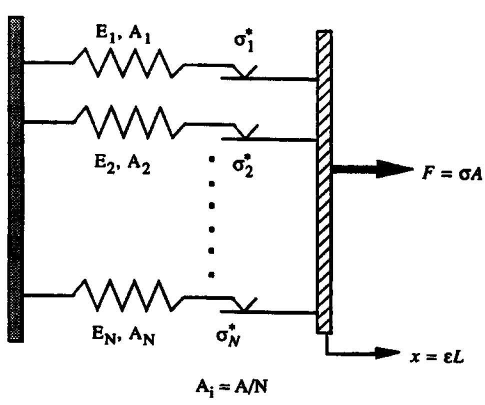

Fig. 1. The Distributed-Element Model for one-dimensional (uniaxial) hysteresis.

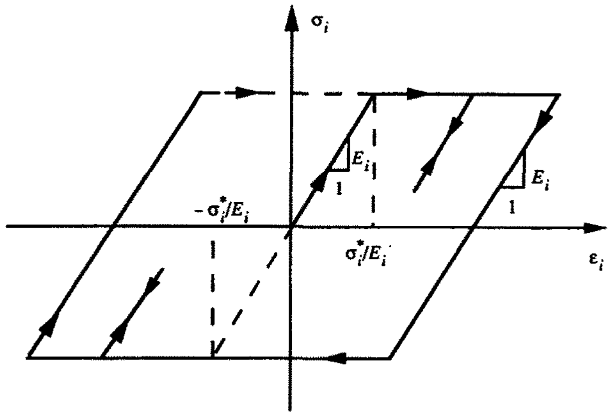

Fig. 2. Hysteretic restoring force behavior of an elasto–perfectly plastic element.

described in Fig. 2. The DEM has been considered as physically motivated, as many real materials or mechanical systems can be thought of as having a similar structure. For example, real materials may have a crystalline structure that is made up of a distribution of slip-planes or dislocations of different slip strengths.

In order to extend the one-dimensional DEMs to multiple dimensions for constitutive modeling of general plasticity, Iwan (1967) introduced the concept of a collection of nested yield surfaces associated with a DEM, which move around in the stress space according to some kinematic rules so that the Bauschinger effect could be accounted for in a more realistic way. This class of multi-dimensional models for plasticity based on the distributed-element formulation provided a conceptual genealization of the customary formulation of the incremental theory of classical plasticity. However, the numerical implementation of such a class of multi-dimensional models involves tracing subsequent yield surfaces and hence is quite difficult and computationally inefficient. With the same motivation, Yoder (1980) proposed an alternative version of plasticity theory formulated in the strain space based on a different class of DEMs from that used by Iwan. Yoder's theory is closely parallel to the traditional theory of plasticity, but interchanges the roles of stress and strain. In contrast to Iwan's multi-dimensional model, the model proposed by Yoder consists of a collection of movable yield surfaces formulated in the *strain* space. However, the same problem arises pertaining to the numerical implementation of the model behavior.

In this study, a new class of constitutive models for plasticity is proposed based on the distributed-element behavior, in which yield surfaces of different yield levels are introduced for the elements involved in a model. The main idea behind this new class of DEMs is that the yield surfaces are defined in the *element*-stress space and are "invariant", i.e. fixed from moving in that space, no matter how the model response varies. Due to the invariant characteristics of the yield surfaces thus defined, the theoretical formulation of such models is so simple that there is no need for any kinematic hardening rules for subsequent yielding behavior. Furthermore, the numerical implementation of the new model is straightforward and highly efficient, even though quite a few elements are needed for the model to yield good results in applications. The validity of this new class of Distributed-Element Models is demonstrated by comparison with experimental results from the literature. It is shown that excellent response predictions using the new models are obtained under complicated multi-axial loading conditions. Another point of interest is that the behavior of this new class of DEMs provides us with a physical model for understanding complicated response mechanisms in cyclic plasticity. This point is examined further in a companion paper (Chiang and Beck, 1993).

2. THEORETICAL FORMULATION

Before looking into the generalization of the one-dimensional (1-D) DEMs to higher dimensions, let us examine in detail the behavior of a 1-D DEM as shown in Fig. 1. It can be shown (Iwan, 1967) that when the number of elements involved in the model becomes very large so that the element stregnths σ_i^* are described in terms of some distribution function $\phi(\sigma^*)$, where $\phi(\sigma^*)\,\mathrm{d}\sigma^*$ denotes the fraction of the total number of elements that have a slip stress between σ^* and $\sigma^*+\mathrm{d}\sigma^*$, then the model behavior can be described by

$$\sigma(\varepsilon) = \int_0^\infty \sigma(\varepsilon, \sigma^*)\phi(\sigma^*)\,\mathrm{d}\sigma^*, \tag{1}$$

where the distribution function $\phi(\sigma^*)$ can be any function that satisfies

$$\int_0^\infty \phi(\sigma^*)\,\mathrm{d}\sigma^* = 1. \tag{2}$$

It has been shown (Jayakumar, 1987) that the 1-D DEMs actually fall within a general class of Masing models whose behavior is described by Masing's hypothesis (Masing, 1926) and some extended rules for hysteresis.

In the following, we will extend the 1-D models to the general multi-dimensional case so that they can be used for constitutive modeling in cyclic plasticity problems. To account for the general multi-axial response behavior, we need to first define the basic kinematic behavior of the distributed elements constituting the model. We postulate the following rules for the new multi-dimensional DEM:

(1) Each element in the model is subject to the same total-strain response as experienced by the model itelf.
(2) Each element has the response behavior of ideal plasticity so that its associated yield surface remains "invariant" in the stress space. In other words, the yield surface associated with an element is described by a function that depends only on the element stress.
(3) All the elements have the same elastic properties and the associated yield functions have the same mathematical form, but they have different yield constants which are governed by some distribution function.
(4) The stress state of the model is defined as the average of the stress states of all the elements. Following these rules, the overall stress of the model can be expressed in terms of the element stress states as follows:

$$\tilde{\sigma}(\tilde{\varepsilon}) \equiv \int_0^\infty \tilde{\sigma}(\tilde{\varepsilon}, k)\phi(k)\,\mathrm{d}k, \tag{3}$$

where $\tilde{\sigma}$ and $\tilde{\varepsilon}$ denote the stress tensor σ_{ij} and the strain tensor ε_{ij}, respectively, and $\tilde{\sigma}(\tilde{\varepsilon}, k)$ is the corresponding stress state of the elements having yielding constant k governed by a distribution function $\phi(k)$. Note the resemblance of eqn (3) to eqn (1). The constant k is related to the yield function associated with each element in the model so that the equation

$$F(\tilde{\sigma}(k), k) = 0 \tag{4}$$

represents a yield surface associated with an element of yield constant k in the element stress space. Note that without loss of generality, we can choose

$$k \equiv \sigma_0(k), \tag{5}$$

where $\sigma_0(k)$ is the uniaxial yield stress of the associated element parametrized by k. The definition of the yield function defined in eqn (4) is conceptually the same as that used in the classical theory of plasticity so as to characterize the general behavior of materials under multi-axial loading conditions. However, in this new formulation, the yield surfaces are defined in the element-stress space, not in the model-stress space as in the classical theory

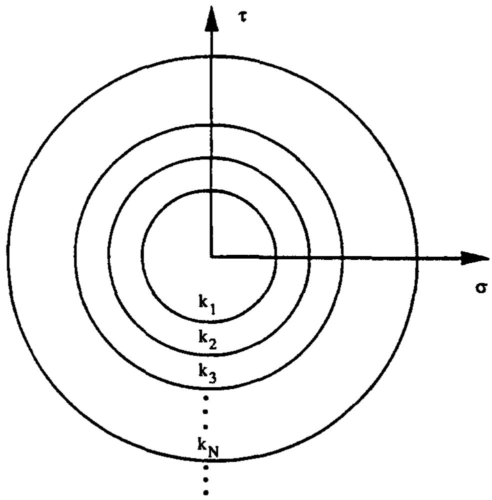

Fig. 3. Invariant yield surfaces nested in the element stress space.

of plasticity. If the element-stress spaces are graphically overlaid, then the yield surfaces are nested within one another due to the different yield strengths of the elements. This is illustrated in Fig. 3, where concentric circles of different radii represent yield surfaces of different yield strengths in the two-dimensional (biaxial) case. Moreover, since each element in the model has the behavior of ideal plasticity, the yield surfaces associated with the elements will remain "invariant" in their space of definition, no matter how the model behaves, so complicated hardening rules for response after initial yielding are avoided. The combined element-stress space is therefore called the "invariant-yield-surface" space. An important remark regarding the overall model behavior is that the stress state of the model may possibly lie outside some of the yield surfaces associated with the elements in the "invariant-yield-surface" space, which makes the new model distinctive from those based on the classical multi-yield-surface theory, by which a model stress state can never lie outside *any* of the yield surfaces. It is also this formulation in the "invariant-yield-surface" space that makes this new model mathematically simple and computationally effective, in contrast to the aforementioned multi-dimensional DEMs proposed by Iwan (1967) and Yoder (1980).

If the yield surface associated with an element with yield constant k is described by eqn (4), then under the assumption of ideal plasticity, we have that when $F(\tilde{\sigma}(k), k) = 0$, plastic flow takes place without limit, and therefore,

$$\mathrm{d}F = \frac{\partial F}{\partial \sigma_{ij}(k)} \mathrm{d}\sigma_{ij}(k) = 0 \tag{6}$$

for plastic flow. From the normality rule of plastic flow given by the classical theory of plasticity, which specifies that the direction of a plastic strain increment is normal to the yield surface at the current stress point, we have the flow rule for an element of yield strength k:

$$\mathrm{d}\varepsilon_{ij}^{p}(k) = \frac{\partial F}{\partial \sigma_{ij}(k)} \mathrm{d}\lambda(k), \tag{7}$$

where $\mathrm{d}\lambda(k)$ is a coefficient of proportionality, whose value can be determined as follows. Firstly, we introduce the general stress–strain relation in incremental form as

$$\mathrm{d}\sigma_{ij}(k) = C_{ijmn}(\mathrm{d}\varepsilon_{mn} - \mathrm{d}\varepsilon_{mn}^{p}(k)), \tag{8}$$

where C_{ijmn} denotes the tensor of elastic constants and it has been assumed that all elements in the model have the same elastic constants and identical total-strain response so that the dependence of C_{ijmn} and $\mathrm{d}\varepsilon_{mn}$ on k can be dropped. Based on eqns (6), (7) and (8), we can find the expression for the coefficient of proportionality as:

$$\mathrm{d}\lambda(k) = \frac{\partial F/\partial\sigma_{ij}(k) C_{ijmn}\,\mathrm{d}\varepsilon_{mn}}{\partial F/\partial\sigma_{pq}(k) C_{pqrs}\,\partial F/\partial\sigma_{rs}(k)}. \tag{9}$$

Summarizing from the above, we arrive at the following set of constitutive equations for the new Distributed-Element Model for general plasticity formulated in the "invariant-yield-surface" space:

$$\tilde{\sigma}(\tilde{\varepsilon}) = \int_0^\infty \tilde{\sigma}(\tilde{\varepsilon}, k)\phi(k)\,\mathrm{d}k, \tag{10a}$$

and

$$F(\tilde{\sigma}(k), k) \leqslant 0 \quad \text{always.} \tag{10b}$$

If

$$F(\tilde{\sigma}(k), k) = 0, \tag{10c}$$

and

$$\mathrm{d}F = \frac{\partial F}{\partial\sigma_{ij}(k)}\,\mathrm{d}\sigma_{ij}(k) = 0, \quad (\text{never} > 0), \tag{10d}$$

then

$$\mathrm{d}\sigma_{ij}(k) = C_{ijmn}\left(\mathrm{d}\varepsilon_{mn} - \frac{\partial F}{\partial\sigma_{mn}(k)}\,\mathrm{d}\lambda(k)\right), \tag{10e}$$

where $\mathrm{d}\lambda(k)$ is given by eqn (8).
If eqn (10c) or (10d) is violated, then

$$\mathrm{d}\sigma_{ij}(k) = C_{ijmn}\,\mathrm{d}\varepsilon_{mn}. \tag{10f}$$

Throughout the above, all the derivatives involving F are to be evaluated at the current value of $\tilde{\sigma}(k)$. Equation (10f) signifies that the instantaneous element response will be linearly elastic if the element is not yielded ($F(\tilde{\sigma}(k), k) < 0$), or if it is yielded but then subject to a condition of unloading ($\mathrm{d}F < 0$).

Through the equations in (10), the model behavior is completely defined as long as the mathematical forms of the two material functions, the yield–strength distribution function $\phi(k)$ and the element yield function $F(\tilde{\sigma}(k), k)$, are specified. The way to define the distribution function $\phi(k)$ is similar to that used for the one-dimensional DEMs, since the general multi-dimensional model should reduce to the one-dimensional case as the loading is restricted to be uniaxial. Thus, similar to eqn (2), the distribution function satisfies

$$\int_0^\infty \phi(k)\,\mathrm{d}k = 1. \tag{11}$$

Also, by eqn (10a), using $\forall k$, $\sigma_{11}(k) = k$ and $\sigma_{ij}(k) = 0$ if $i \neq 1$ or $j \neq 1$ (which signifies that every element is in yielding state under the uniaxial loading condition), we have

$$\sigma_u = \int_0^\infty k\phi(k)\,\mathrm{d}k, \tag{12}$$

where σ_u denotes the ultimate uniaxial stress of the model. Equations (11) and (12) provide two conditions for the yield–strength distribution function $\phi(k)$ to satisfy. Therefore, $\phi(k)$ can be chosen initially as any probability density function that has the mean value σ_u as a parameter, and then the resulting response of the DEM can be studied to examine the consequences of this choice. To this end, the Rayleigh distribution, defined as

$$\phi(k) = \frac{\pi}{2}\frac{k}{\sigma_u^2}\exp\left(\frac{-\pi}{4}\frac{k^2}{\sigma_u^2}\right), \tag{13}$$

serves as a good candidate for $\phi(k)$ due to its analytically-tractable form. Additional parameters may be incorporated in the definition of $\phi(k)$, if desired, so that more general yielding behavior can be modeled (Chiang, 1992).

As for the yield function of the elements, there have been numerous yield criteria proposed in plasticity theory for various materials. Among them, the von Mises yield criterion, described by [using (5)]:

$$F(\tilde{\sigma}(k), k) \equiv \tfrac{1}{2}s_{ij}(k)s_{ij}(k) - \tfrac{1}{3}k^2 = 0 \tag{14}$$

is probably the most widely recognized criterion for modeling yielding behavior of materials due to its physical consistency and mathematical tractability. In eqn (14), s_{ij} denotes the deviatoric stress tensor defined as

$$s_{ij} = \sigma_{ij} - \tfrac{1}{3}\sigma_{mm}\delta_{ij},$$

where δ_{ij} is the Kronecker delta function. Another possibility is Tresca's criterion which has been used extensively in the classical theory of plasticity (Mendelson, 1968). Of course, the yield function can be chosen appropriately for the system under consideration based on any criterion used in plasticity theory or even based on an empirical condition identified experimentally.

It is not difficult to show that the elastic constants, assumed to be identical for all elements, are essentially the same as those of the model itself. This property makes the modeling of this class of multi-dimensional DEMs very straightforward, since the elastic constants associated with various materials have been well documented, or can be found through simple experiments. It should be pointed out that although we assume that all the elements in a DEM are subject to the same total strain increment as experienced by the model itself, i.e. $d\varepsilon_{ij}(k) = d\varepsilon_{ij}$, $\forall k$, the plastic strain response of the model is given by

$$d\varepsilon_{ij}^{p} = \int_0^\infty d\varepsilon_{ij}^{p}(k)\phi(k)\,dk, \tag{15}$$

as can be derived using eqns (3) and (8), where $d\varepsilon_{ij}^{p}(k)$ is to be found from eqns (7) and (9), and it is different for each element, in general.

In summary, this class of DEMs formulated in the "invariant-yield-surface" space for cyclic plasticity involves only very few parameters that have clear physical significance. In the case where isotropic materials are of interest, if the Rayleigh distribution is used for the yield–strength distribution function $\phi(k)$, only three parameters: uniaxial peak stress σ_u, Young's modulus E and Poisson's ratio ν, are sufficient to represent realistic multi-axial elastic–plastic response behavior. The modeling process or the identification procedure for the new DEM is therefore simple and efficient.

3. NUMERICAL IMPLEMENTATION

In theory, a Distributed-Element Model may consist of an infinite number of elements whose yield strengths are distributed according to the specified distribution function $\phi(k)$, and the model response is found by keeping track of each element's behavior [cf. eqn (10)]. However, to numerically implement the formulation, one has to restrict the model to a finite number of elements. In order to preserve the advantages of this simple, physical model, it is proposed that the introduction of the finite number of elements be made according to the specified yield-strength distribution function $\phi(k)$, so that the number of parameters involved in the model does not increase with the number of elements introduced. In the case where the Distributed-Element Model consists of a finite number of, say N, elements, the integral operation in eqn (10a) is replaced by the summation operation as follows:

$$\tilde{\sigma}(\tilde{\varepsilon}) = \sum_{i=1}^{N} \tilde{\sigma}(\tilde{\varepsilon}, k_i)\psi(k_i), \tag{16}$$

where the "weighting function" $\psi(k_i)$ satisfies

$$\sum_{i=1}^{N} \psi(k_i) = 1, \tag{17}$$

in place of eqn (11). Also eqn (12) becomes

$$\sum_{i=1}^{N} k_i\psi(k_i) = \sigma_u. \tag{18}$$

In order to obtain smooth response curves, one can choose, without loss of generality,

$$\psi(k_i) = \frac{1}{N} \quad \forall\, i = 1, \dots, N, \tag{19}$$

and the yield constants k_i, $i = 1, \dots, N$, are selected based on the specified distribution function $\phi(k)$, $k \in [0, \infty)$, so that each time a new element yields, the model loses 1/N of its initial stiffness. This can be done by dividing the region below the curve described by the distribution function into N equal-area portions, and selecting k_i as a representative value for the ith portion, so that eqns (18) and (19) are satisfied, that is

$$\sum_{i=1}^{N} k_i = N\sigma_u. \tag{20}$$

The aforementioned modeling procedure is illustrated schematically in Fig. 4. For most applications, it suffices to use 10 elements or so in representing the new model in order to get a reasonably smooth hysteresis curve. For example, the yield constants for the 10 elements corresponding to a Rayleigh distribution can be defined as:

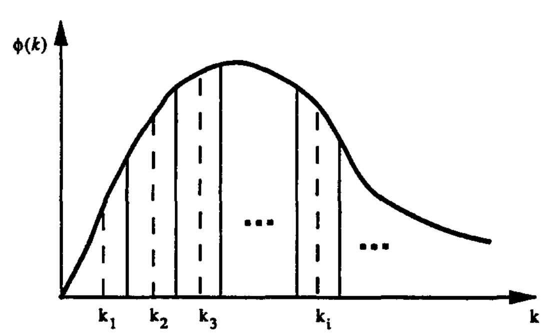

Fig. 4. Selection of yield constants for a finite number of elements according to the specified strength distribution function.

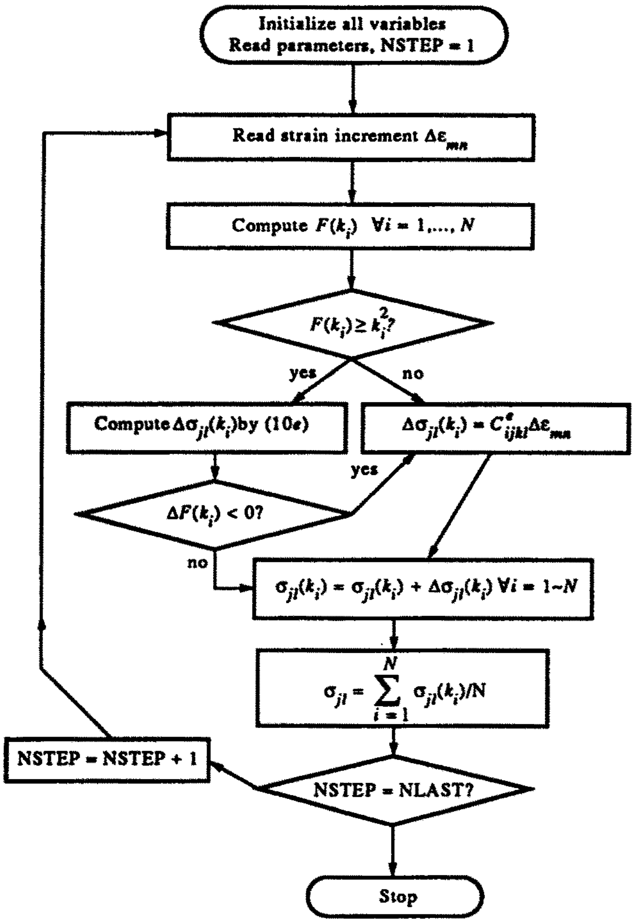

Fig. 5. A flow diagram showing numerical procedure for obtaining stress response of an N-element DEM.

$$\bar{k}_1 = 0.2638, \quad \bar{k}_2 = 0.4601, \quad \bar{k}_3 = 0.6097, \quad \bar{k}_4 = 0.7448, \quad \bar{k}_5 = 0.8767,$$
$$\bar{k}_6 = 1.0128, \quad \bar{k}_7 = 1.1612, \quad \bar{k}_8 = 1.3347, \quad \bar{k}_9 = 1.5630, \quad \bar{k}_{10} = 1.9732, \tag{21}$$

where we define $\bar{k}_i \equiv k_i/\sigma_u$.

The numerical procedure for obtaining the stress response of an N-element model based on the invariant-yield-surface formulation, subject to some prescribed strain path, can be best described by a flow diagam as shown in Fig. 5, where we assume that the strain increment in each loading step is small; otherwise, some subdivision of $\Delta\varepsilon$ is needed to accurately monitor whether or not eqns (10c) and (10d) are satisfied. As can be seen from the flow diagram in Fig. 5, the numerical implementation of this new multi-dimensional class of DEMs is surprisingly simple and computationally efficient. This is due to the formulation in the invariant-yield-surface space, which avoids the usually complicated hardening rules required for accounting for the Bauschinger effect in cyclic plasticity.

4. AN APPLICATION TO BIAXIAL LOADING

Simulation studies on the response of the new Distributed-Element Model to prescribed strain paths are conducted to examine the model behavior in the biaxial tension–torsion case, for which published work is readily available for comparison. Lamba and Sidebottom (1978a, b) conducted a series of biaxial tension–torsion tests on copper in which cyclic, nonproportional axial-torsional strain paths were applied to examine material response behavior. The test examples used were thin-walled hollow cylindrical shafts and were loaded with combined axial force and torsion, so that the tensors of stress and strain are uniform in space and can be represented approximately as

$$\sigma_{ij} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & 0 \\ \sigma_{12} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \varepsilon_{ij} = \begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} & 0 \\ \varepsilon_{12} & -\gamma\varepsilon_{11} & 0 \\ 0 & 0 & -\gamma\varepsilon_{11} \end{pmatrix}, \tag{22}$$

where the coefficient γ represents the Poisson effect which is a variable when inelastic deformations are involved in the response. It can be shown that if we assume incompressibility of plastic deformation, for example, then we have the following approximate expression for γ:

$$\gamma = \frac{1}{2} - \frac{1}{E}\left(\frac{1}{2} - \nu\right)\frac{d\sigma_{11}}{d\varepsilon_{11}}, \tag{23}$$

where ν is the Poisson's ratio for linear elasticity.

In the simulation studies, the model used consists of 10 distributed elements, and the Rayleigh distribution is used for describing the yield–strength distribution in the formulation, so that eqn (21) defines the yield constants of the elements. The model parameters used are $E = 16{,}700$ ksi, $\nu = 0.33$, and $\sigma_u = 30$ ksi, which are taken directly from the experimental results. The prescribed strain loading paths are shown in Fig. 6, for which the corresponding experimentally-observed stress responses are available (Lamba and Sidebottom, 1987a, b), as shown in Figs 7 and 8. Note that the loading path sequence in Fig. 6(a) is 0-1-0-2-0-3-0-..., so as to study the property of erasure-of-memory, which will be elucidated in more detail in Part II of this study. Also, the stress path resulting from the repetition of path 0 each time is not plotted in Fig. 7(a) for clarity.

The stress responses predicted using the DEMs are shown, respectively, in Figs 9 and 10, where both von Mises' and Tresca's yield criteria were used in the simulations for comparison purposes. In general, the results obtained in all cases are both qualitatively and quantitatively consistent with those observed experimentally, and Tresca's yield condition gives slightly better results than von Mises' does considering the value of the ultimate shear stress predicted. Note that Fig. 9(a) contains the full stress path whereas Fig. 7(a) does not. It can be clearly seen in Figs 7 and 8, that there exists equilibrium points corresponding to uni-directional strain paths, at which the stress increments approach zero for appreciable strain increments. In addition, it is suggested from the experimental results and the model predictions that there exists a limit surface in the stress space in each of the two loading cases beyond which stress states never go. Moreover, an "erasure-of-memory" property (Lamba and Sidebottom, 1978a, b) is clearly demonstrated by the DEM, as one can see that the model is always brought back to the same stress state every time the path 0 in Fig. 6(a) is traced. This is in good agreement with the experimental results. In Part II of this study (Chiang and Beck, 1993), the issues of the existence and uniqueness of equilibrium points and those of the limit surface will be addressed in detail based on the behavior of the new class of DEMs.

Other important response features in cyclic plasticity, such as smooth yielding, nonlinear strain hardening and multi-axial Bauschinger effect are also well demonstrated by the new DEM. The computational effort involved in obtaining the response based on the new model is small, since no kinematic hardening rule is required to account for the subsequent yielding behavior of materials. This makes the new model suitable for response analysis of complex structural systems using the finite-element method. We remark that models based on the classical theory of plasticity in general do not predict response behavior as well as the DEMs do, as we can see in Fig. 11, where different yield conditions together with different kinematic hardening rules were employed to predict the response to the strain loading path given in Fig. 6(b). The dashed curve in each plot of Fig. 11 represents the

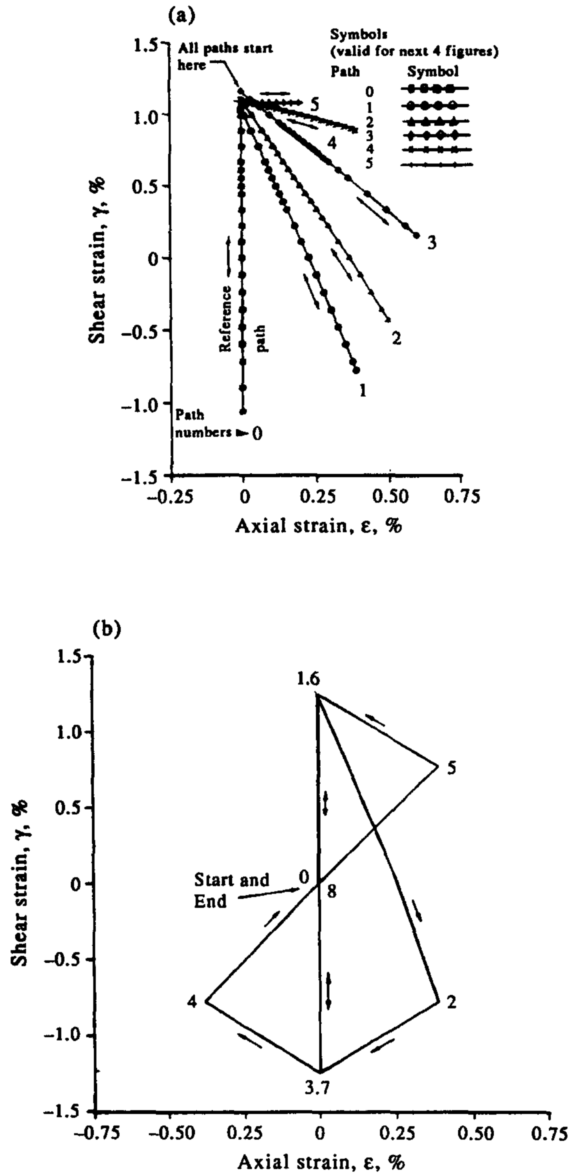

Fig. 6. Prescribed strain loading paths for response studies of the proposed multi-dimensional DEMs [from Lamba and Sidebottom (1978)].

locus of the center of the yield surface, which is irrelevant to the discussion here. A clear deficiency of the first two models, which use, respectively, the von Mises yield condition with the Prager hardening rule and the Tresca yield condition with the Ziegler hardening rule, is that the predicted axial stress does not return to zero. These two models also fail to demonstrate the behavior of equilibrium points and a limit surface exhibited by real materials. In Fig. 11(c), a much more elaborated model is used, which employs a Tresca yield surface, a Tresca limit surface, together with the Mroz kinematic hardening rule (Mroz, 1967), and an empirical nonlinear strain hardening assumption (Lamba and Sidebottom, 1978b), so as to give the plasticity model a maximum chance of success.

It should be noted that in the preceding examples, we did not use any system identification technique to optimally choose the model parameters by fitting the stress and strain histories. Instead, the values of the parameters E and ν were those specified by Lamba and Sidebottom and σ_u was read directly from the corresponding biaxial test curves. This advantage is obviously due to the physical consistency and the parsimony in parameters of the

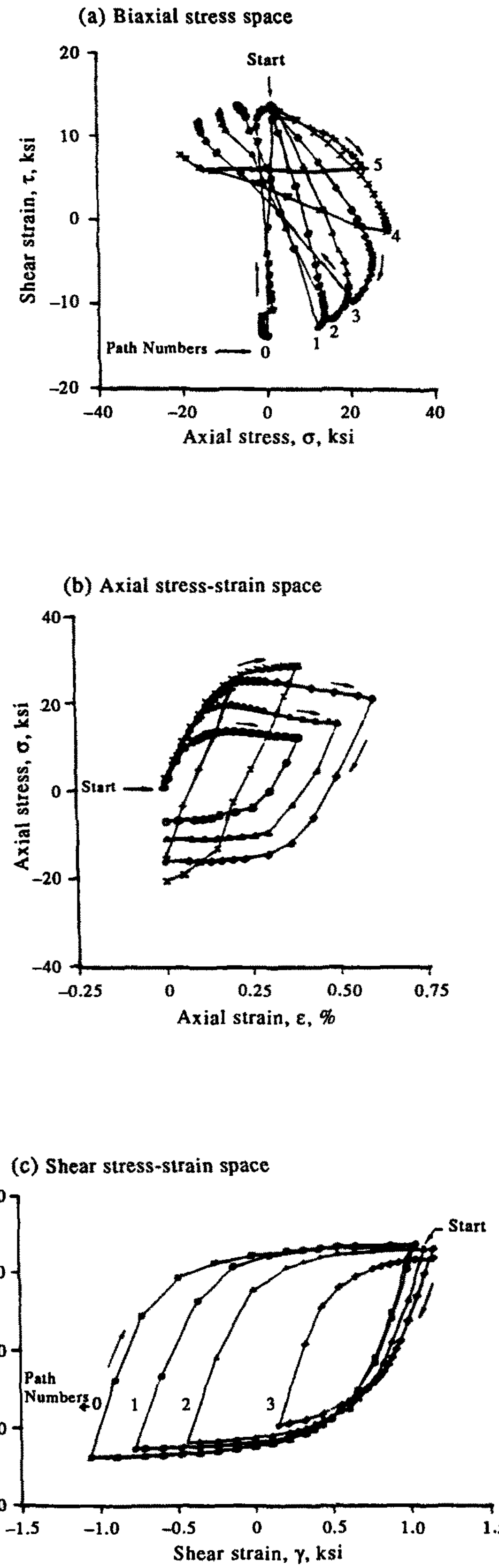

Fig. 7. Experimentally-observed stress response of copper to the prescribed strain path given in Fig. 6(a) [from Lamba and Sidebottom (1978)]: (a) biaxial stress space; (b) axial stress–strain space; (c) shear stress–strain space.

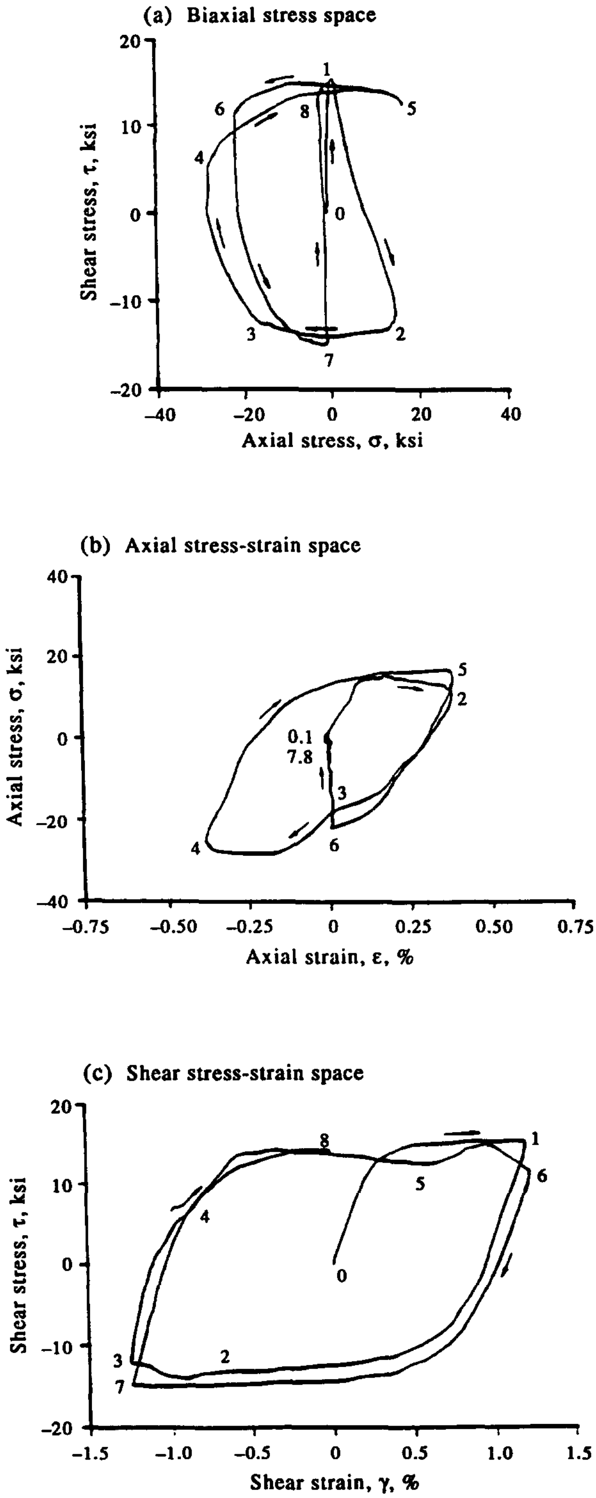

Fig. 8. Experimentally-observed stress response of copper to the prescribed strain path given in Fig. 6(b) [from Lamba and Sidebottom (1978)]. (a) biaxial stress space; (b) axial stress–strain space; (c) shear stress–strain space.

proposed DEM for cyclic plasticity. In the case where complex structural systems are of interest, the parameters may be optimally identified using structural response data. Furthermore, we can treat constants in the yield condition required for the model as parameters to be identified, so that the "best" result may be achieved in practice.

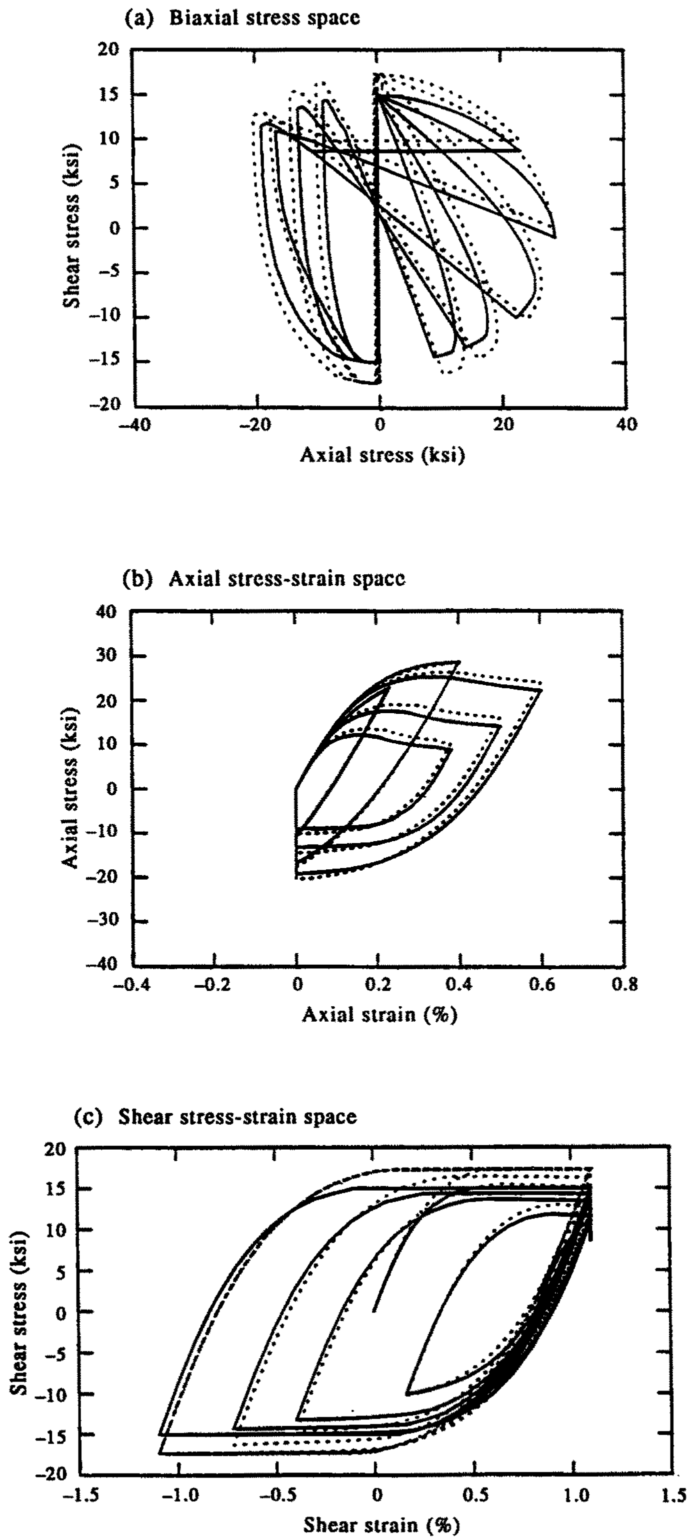

Fig. 9. Stress response predicted by a new DEM subject to the prescribed strain path given in Fig. 6(a) (Tresca ——, von Mises ----) : (a) biaxial stress space; (b) axial stress–strain space; (c) shear stress–strain space.

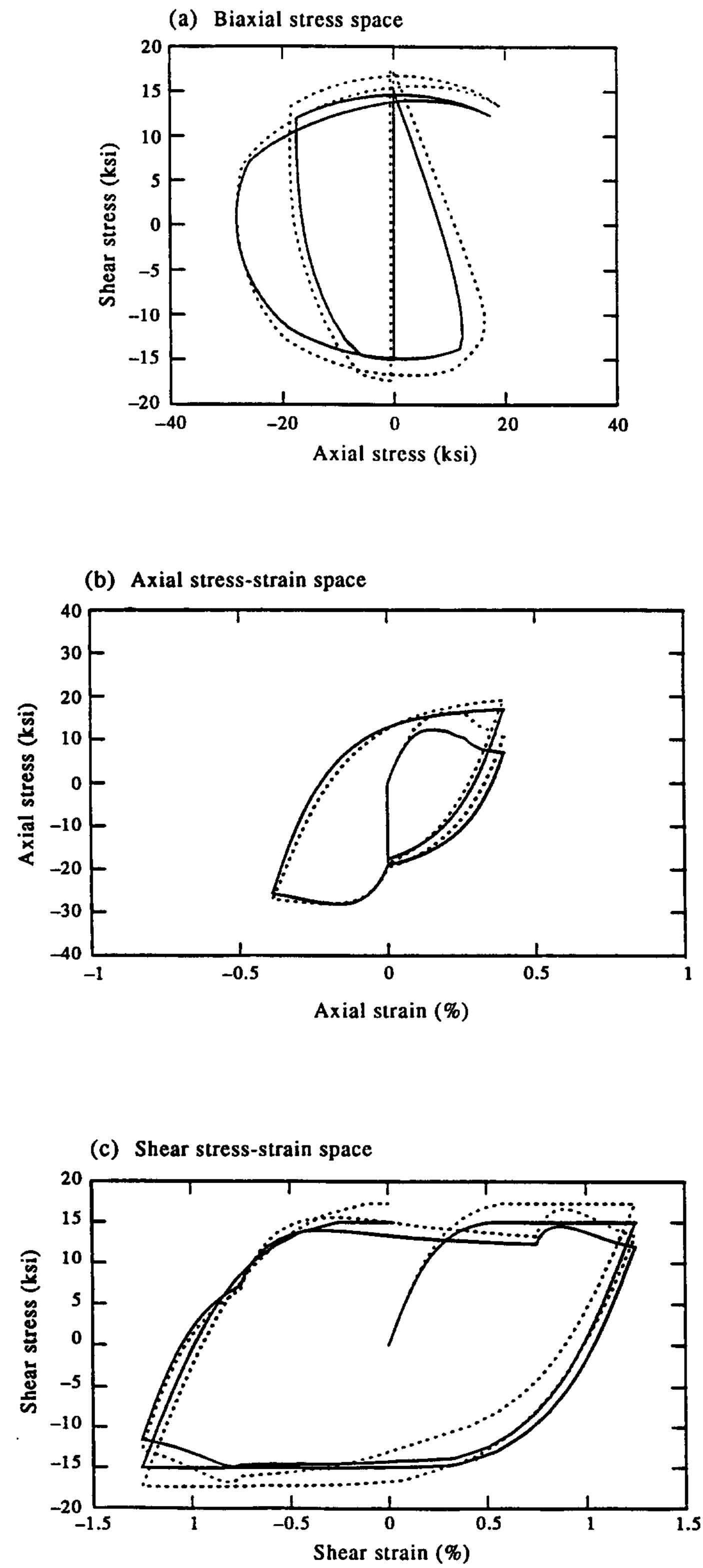

Fig. 10. Stress response predicted by a new DEM subject to the prescribed strain path given in Fig. 6(b) (Tresca ——, von Mises ----) : (a) biaxial stress space; (b) axial stress–strain space; (c) shear stress–strain space.

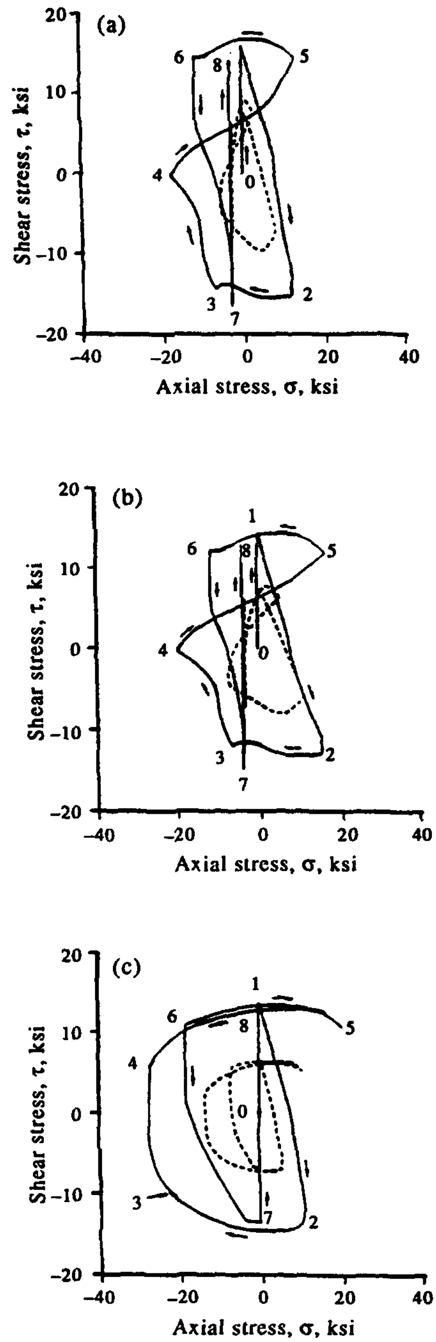

Fig. 11. Response predicted by different models of plasticity to the strain path given in Fig. 6(b): (a) von Mises' yield surface with Prager's hardening rule; (b) Tresca's yield surface with Ziegler's hardening rule; (c) Tresca's yield surface and limit surface with Mroz' hardening rule [from Lamba and Sidebottom (1978)].

5. CONCLUSIONS

A new class of Distributed-Element Models is proposed for constitutive modeling in cyclic plasticity. In the theory, a formulation in the "invariant-yield-surface" space is presented so that this physically-motivated model is not only mathematically simple and computationally efficient but also parsimonious in parameters. The physical consistency of this new class of Distributed-Element Models was also demonstrated by comparison with experimentally-observed results, in which excellent response predictions using the new models were obtained under nonproportional biaxial loading conditions.

We remark that the new Distributed-Element Model for multi-dimensional plasticity could be viewed as a statistical mechanical model which is a generalization of the classical formulation of plasticity theory. In the new theory, the yield condition for elastic–plastic response characterization and the flow rule for prescribing plastic strain increments are treated in a statistical sense, so that the model response is the statistical average of the element response, each of which follows from the classical theory of plasticity. The elements could be viewed physically as a distribution of slip-planes or dislocations of different slip strengths in a small volume of the material at a point of interest.

In Part II of this study (Chiang and Beck, 1993), we will address some important properties associated with the new class of DEMs for general plasticity. Thorough understanding of these properties helps to explain some material properties and complicated material behavior under cyclic multi-axial loading conditions.

REFERENCES

Chiang, D. Y. (1992). Parsimonious modeling of inelastic systems. Report No. EERL 92-02, California Institute of Technology, Pasadena.

Chiang, D. Y. and Beck, J. L. (1994). A new class of DEM for cyclic plasticity—II. On important properties of material behavior. *Int. J. Solids Structures* **31**(4), 485–496.

Iwan, W. D. (1966). A distributed element model for hysteresis and its steady-state dynamic response. *J. Appl. Mech. ASME* **33**(4), 893–900.

Iwan, W. D. (1967). On a class of models for the yielding behavior of continuous and composite systems. *J. Appl. Mech. ASME* **34**(2), 612–617.

Jayakumar, P. (1987). Modeling and identification in structural dynamics. Report No. EERL 87-01, California Institute of Technology, Pasadena.

Lamba, H. S. and Sidebottom, O. M. (1978a). Cyclic plasticity for nonproportional paths—I. Cyclic hardening, erasure of memory, and subsequent strain hardening experiments. *J. Engng Mater. & Tech.* **100**, 96–103.

Lamba, H. S. and Sidebottom, O. M. (1978b). Cyclic plasticity for nonproportional paths—II. Comparison with predictions of three incremental plasticity models. *J. Engng Mater. & Tech.* **100**, 104–111.

Masing, G. (1926). Eigenspannungen und verfestigung beim messing (self stretching and hardening for brass). *Proc. 2nd International Congress for Applied Mechanics*, Zurich, Switzerland, pp. 332–335 (in German).

Mendelson, A. (1968). *Plasticity: Theory and Application.* MacMillan, New York.

Mroz, Z. (1967). On the description of anisotropic work hardening. *J. Mech. Phys. Solids* **15**, 163–175.

Yoder, P. J. (1980). A strain–space plasticity theory and numerical implementation. Report No. EERL 80-07, California Institute of Technology, Pasadena.